Asset Pricing: An Estimation Alternative to the Five-Factor Model

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Abbreviated abstract: The five factors are summarized by the risk premium, the size of companies, the Book-to-Market ratio, profitability, and investment. In this way and focusing on the Brazilian stock market, this paper aims to analyze the impact of the risk premium on the excess returns from several portfolios under several specifications. Considering that the distributional behavior of the estimated models shows evidence against the Gaussian pattern, we proposed the usage of the GAMLSS (Generalized Additive Models for Location, Scale, and Shape), which allow exploring several random patterns to stock returns.

Problem, Portfolio Analysis and Factors' Risk

- In this paper we modeled excess returns in the Brazilian financial market using a broad class of models that encompasses a large number of probability laws: the GAMLSS class.
- In particular, we investigate the role played by risk premiums in the five-factor model.
- We show that models based on the Gaussian law do not fully capture the data dynamics and, as a result, display evidence of model misspecification.

Data and portfolio analysis

Portfolios *size* × *B/M*		*B/M*		
		Low (L)	Neutral (N)	High (H)
size	Small (S)	SL	SN	SH
	Big (B)	BL	BN	BH

Portfolios *size* × *OP*		*OP*		
		Weak (W)	Neutral (N)	Robust (R)
size	Small (S)	SW	SN	SR
	Big (B)	BW	BN	BR

Portfolios *size* × *INV*		*INV*		
		Conservative (C)	Neutral (N)	Aggressive (A)
size	Small (S)	SC	SN	SA
	Big (B)	BC	BN	BA

The factors' risk premiums are described bellow:

- CAPM
- SMB (small minus big);
- HML (high minus low B/M);
- RMW (robust minus weak OP);
- CMA (conservative minus aggressive INV)

Methods

The risk premiums of the five factors are computed using the portfolios assembled based on monthly returns.

$$R_{i,t} - R_{f,t} = a_i + b_i(R_{m,t} - R_{f,t}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{i,t}.$$

- GAMLSS models are semiparametric regression models that require a parametric distribution assumption for the response variable.
- The GAMLSS framework includes a wide range of statistical distributions that can be used to model the variable of interest, including continuous and discrete distributions with different levels of asymmetry and kurtosis.
- Let $y_1, \ldots, y_T$ be independent variables, each y_t, for $t = 1, \ldots, T$, having probability density function $f(y_t|\theta^t)$, where $\theta^t = (\theta_{1t}, \theta_{2t}, \theta_{3t}, \theta_{4t}) = (\mu_t, \sigma_t, \nu_t, \tau_t)$.

The vector of parameters β_k in Equation

$$g_k(\theta_k) = \eta_k = X_k\beta_k, k = 1,..4.$$

are estimated within the GAMLSS framework by maximizing the penalized log-likelihood function which for our proposal is simplified by $l_p = \sum_{i=1}^{n} \log f(y_i/\theta^i) - \frac{1}{2}\sum_{k=1}^{p}\sum_{j=1}^{j_k} \lambda_{jk}\gamma'_{jk}G_{jk}\gamma_{jk}$,

Results and Conclusions

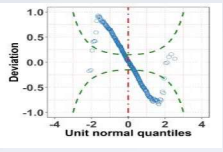
Table: Parameter estimates and z test statistics, location (μ) GAMLSS model, combinations of size-B/M, size-OP and size-INV. The symbol '$*$' is indicative of significant at 5%. Values in parentheses indicate standard error.

Regression equation: $R_t - R_{f,t} = a + b(R_{m,t} - R_{f,t}) + sSMB_t + hHMLM_t + rRMW_t + cCMA_t$

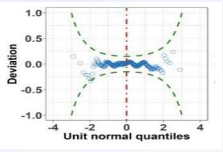
Models estimates

Size/ Factor	*B/M*			*OP*			*INV*		
(Level)	Low (L)	Neutral (N)	High (H)	Weak (W)	Neutral (N)	Robust (R)	Conservative (C)	Neutral (N)	Aggressive (A)
(Parameter)		*a*			*a*			*a*	
Small (S)	0.006(0.005)	−0.008*(0.004)	−0.006(0.004)	−0.005(0.002)	−0.003(0.004)	0.006(0.004)	−0.007(0.004)	−0.008(0.004)	−0.0003(0.004)
Big (B)	−0.010*(0.003)	0.003(0.003)	−0.0003(0.004)	−0.0004(0.004)	−0.00001(0.003)	−0.006*(0.003)	0.005(0.004)	−0.005*(0.002)	−0.005(0.004)
		b			*b*			*b*	
Small (S)	1.009*(0.028)	0.930*(0.031)	0.898*(0.023)	0.908*(0.017)	0.918*(0.029)	0.957*(0.025)	0.966*(0.025)	0.917*(0.024)	0.936*(0.028)
Big (B)	0.943*(0.021)	0.935*(0.025)	0.967*(0.020)	0.958*(0.026)	0.859*(0.031)	0.938*(0.020)	0.897*(0.030)	0.951*(0.002)	0.926*(0.027)
		s			*s*			*s*	
Small (S)	1.246*(0.050)	0.614*(0.050)	0.816*(0.035)	0.969*(0.021)	0.710*(0.067)	0.686*(0.077)	0.898*(0.029)	0.773*(0.041)	0.845*(0.053)
Big (B)	−0.156*(0.035)	−0.097*(0.039)	−0.088*(0.020)	−0.238*(0.042)	−0.137*(0.017)	0.060(0.052)	−0.037(0.063)	0.059*(0.024)	0.022(0.013)
		h			*h*			*h*	
Small (S)	−0.893*(0.071)	0.076(0.045)	0.431*(0.037)	−0.147*(0.025)	−0.019(0.041)	0.148*(0.016)	−0.121*(0.034)	0.200*(0.025)	0.061(0.045)
Big (B)	−0.283*(0.031)	−0.024(0.029)	0.459*(0.026)	0.085*(0.038)	0.041(0.043)	−0.112*(0.025)	−0.074*(0.027)	−0.076*(0.019)	−0.322*(0.026)
		r			*r*			*r*	
Small (S)	0.330*(0.054)	0.098*(0.037)	−0.134*(0.019)	−0.667*(0.025)	0.000(0.024)	0.346*(0.011)	−0.196*(0.025)	−0.096*(0.020)	−0.051(0.039)
Big (B)	0.121*(0.024)	−0.154*(0.020)	−0.280*(0.027)	−0.606*(0.034)	−0.074*(0.026)	0.201*(0.022)	−0.058*(0.027)	0.107*(0.020)	−0.153*(0.017)
		c			*c*			*c*	
Small (S)	−0.176*(0.045)	0.018(0.024)	0.072*(0.018)	0.037*(0.008)	0.033(0.022)	−0.024*(0.011)	0.417*(0.018)	−0.092*(0.014)	−0.498*(0.043)
Big (B)	−0.176*(0.028)	−0.071*(0.015)	0.074*(0.012)	0.074*(0.018)	−0.325*(0.017)	0.010(0.015)	0.341*(0.044)	−0.080*(0.014)	−0.603*(0.010)

Worm-Plot: Standard Linear Regression



Worm-Plot: GAMLSS



- Using the data-selected laws, we fitted GAMLSS models for the location (mean) parameter which we called 'best fitting GAMLSS models'.
- The diagnostic plots showed no evidence of incorrect model specification for such models.
- This is indicative that excess returns in the Brazilian financial data do not follow the Gaussian law, and that models that use it as the underlying data generating process may fail to capture the true data dynamics.