Title: **An Optical Flow computation based on Shearlet Transform: A mathematical Formulation**

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Abbreviated abstract: Optical flow calculates frame-to-frame motion. Robotics, video, and space use it. Horn–Schunck, Lucas–Kanade, Farneback, and wavelet transform-based optical flows are realized. Integrating the wavelet transform solved the optical flow's limited capture range and low precision (despite significant displacement). Edges, curves, and multidimensional singularities challenge wavelets. Shearlet transform efficiently handles image and video singularities. This study integrates the shearlet transform and optical flow technique in a mathematical formulation. This is to achieve robust optical flow estimation.

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Problem, Data, Previous Works

Problem Definition

Wavelets are deficient in dealing with multidimensional and multivariate data such as edges, contours and multidimensional singularities.

Review of related works

Zheng, J., et al., (2019):

- ❑ multi-resolution wavelet transform approach was adopted
- ❑ However, isotropic Gaussian filtering used in the wavelet approaches gave a blurry output and inaccurate detection of boundary orientation in the presence of sharp changes in the curvature

MPI Sintel Dataset

- ❑ Provides a dense groundtruth derived from the open-source 3D animated short film Sintel

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Methods

The optical flow constraint equation can be expressed in equation (2.20) (Zheng et al., 2019):

$$I(x, y, t) = I(x + \Delta x, y + \Delta y, t + \Delta t) \qquad (1)$$

Assuming small and approximate constant movement between image frames, the Taylor's series expansion of equation (1) result in equation (2):

$$I(x + \Delta x, y + \Delta y, t + \Delta t) = I(x, y, t) + \frac{\partial I}{\partial x}\Delta x + \frac{\partial I}{\partial y}\Delta y + \frac{\partial I}{\partial t}\Delta t + H.O.T \qquad (2)$$

$$Qv = X \qquad (3)$$

The discrete shearlet basis can be defined as presented in equation (4) (S. Singh et al., 2015):

$$\psi_{j,k,m}^{n}(x, y) = 2^{j\frac{3}{2}}\psi^{n}(S_n^k D_n^k x - m_1, S_n^k D_n^k y - m_2) \qquad (4)$$

The inner product of equations (2) and (4) results in equation (5)

$$\langle \frac{\partial I}{\partial x} V_x, \psi_{j,k,m}^{n} \rangle + \langle \frac{\partial I}{\partial y} V_y, \psi_{j,k,m}^{n} \rangle + \langle \frac{\partial I}{\partial t}, \psi_{j,k,m}^{n} \rangle = 0 \qquad \forall_n = 1 \ldots N \qquad (5)$$

Substituting matrix parameters of equations

$$v_m^{jk} = ((Q_m^{jk})^T Q_m^{jk})^{-1} (Q_m^{jk})^T X_m^{jk} \qquad (6)$$

Where,

Q_m^{jk} denotes the system matrix

X_m^{jk} is the observation matrix

v_m^{jk} is the affine parameter matrix

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Conclusions

In conclusion, this research will adopt ST due to its numerous advantages, including anisotropic directional decomposition, computational stability and efficiency, and scale and translation invariances. Consequently, a more stable, computationally efficient, informative, and clearer optical flow output will be achieved.

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