Title: A modification to the Fuzzy Regression Discontinuity model

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Abstract:

Despite the fact that fuzzy regression discontinuity designs are growing in popularity, a lot of research takes into account treatment non-compliance difficulties, specifically the fuzziness of the treatment impact. The modified Fuzzy Regression Discontinuity model is preferable for modeling fuzzy data. Once the data is crisp, the traditional least squares methods of approximation are used to estimate the parameters in the model since these parameters are considered crisp whilst the error terms are fuzzy.

Related publications:

Zhang, A. et al. (2016). Statistical analysis of fuzzy linear regression model based on centroid method. Applied Mathematics, 7(07):579.

Sasabuchi, Y. (2022). Introduction to regression discontinuity design. Annals of Clinical Epidemiology, 4(1):1–5.

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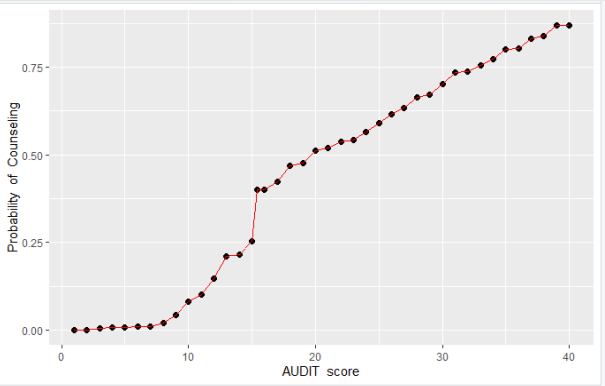
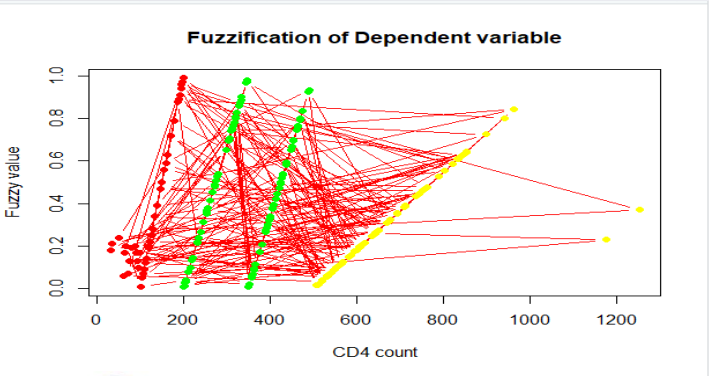
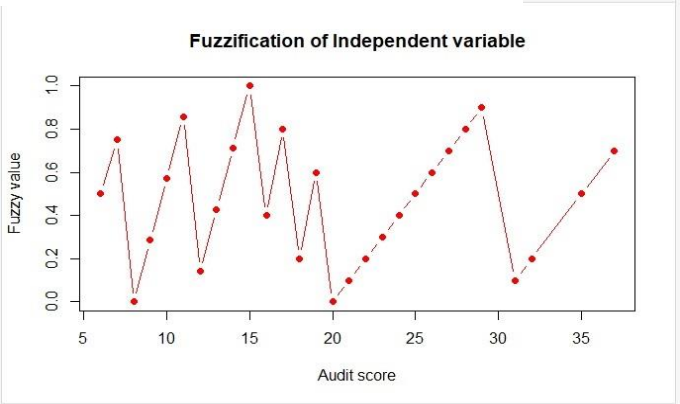
Problem statement

- The approach that is considered fuzzy has received the majority of attention in research on fuzzy regression discontinuity models. In the same model, fuzzy dependent and independent variables have received little to no attention.
- The studied model consists of crisp variables and is given by $y_{ij} = \alpha + \beta(1 - d_{ij})(x_{ij} - x') + \gamma d_{ij}(x_{ij} - x') + \delta d_{ij} + e_{ij}$







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Proposed Model

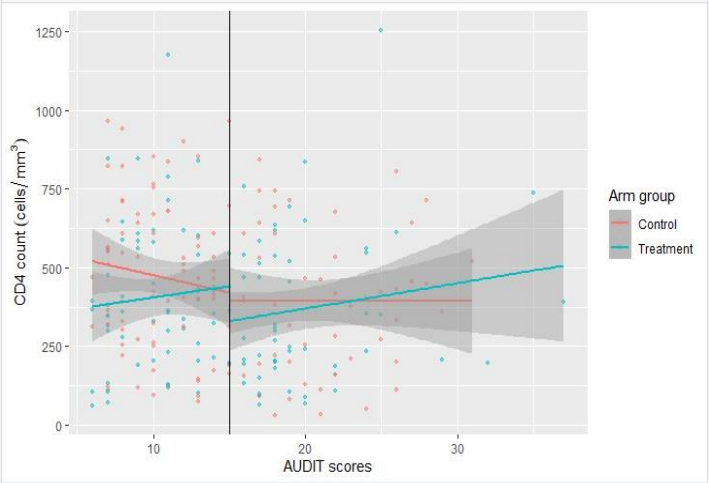
- The proposed model is given by $y_{ij}^{*} = \alpha + \beta(1 - \hat{d_{ij}})(x_{ij}^{*} - x'^{*}) + \gamma\hat{d_{ij}}(x_{ij}^{*} - x'^{*}) + \delta\hat{d_{ij}} + e_{ij}^{**}$ $\mu_{y_{ij}}(y)$ and $\mu_{x_{ij}}(x)$ respectively.

- Moreover, e_{ij}^{**} is the fuzzy error associated with the model and the parameters of the model are crisp.

- To estimate the fuzzy error terms, e_{ij}^{**} we sticked to the idea from (Kim and Bishu, 1998). That is to use the ratio of the difference in membership va

$$e_{ij}^{**} = \frac{\int_{S_{y_{ij}} \cup S_{\hat{y}_{ij}}} |\mu_{y_{ij}}(y) - \mu_{\hat{y}_{ij}}(y)| dy}{\int_{S_{y_{ij}}} \mu_{y_{ij}}(y) dy}$$

- The figure shows the representation of the Modified Fuzzy Regression Discontinuity as the treatment effect is also fuzzy.



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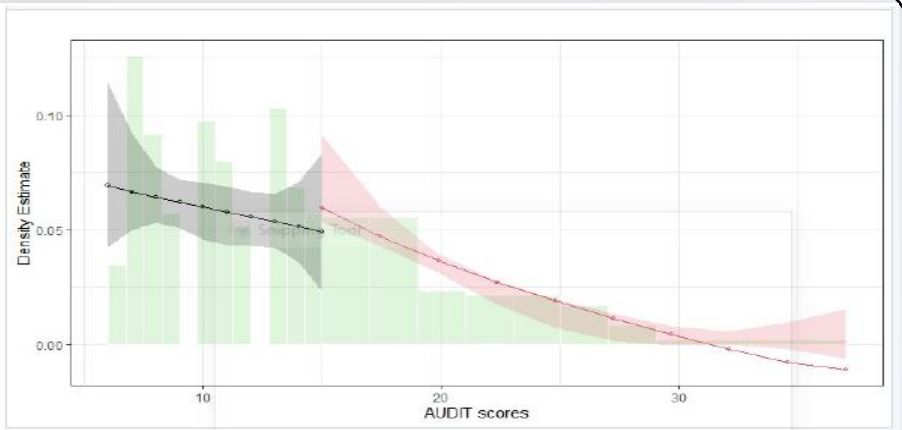
Results and Conclusions

- The overlap's p-value is 0.465517, which is significantly greater than 0.05. The McCrary test as shown on the figure demonstrates with certainty that there is no manipulation or bunching.
- Our approach accommodates cases in which the variables are fuzzy.
- The table shows that counseling PLWHA who consume alcohol leads to a causal effect of approximately 0.237 in the positive direction.



Parametric Estimation: Fuzzy RDD

term	estimate	standard error	statistic	p.value	conf.low	conf.high
(Intercept)	10.24623623	0.60648725	16.8943966	4.01E-42	9.0512275	11.4412449
audit_centered	0.05264287	0.08106509	0.6493901	5.17E-01	-0.1070859	0.2123717
counsellingTRUE	0.23715658	1.00934232	0.2151466	8.30E-01	-1.7716286	2.2059418

- Sensitivity testing for adherence rate of 90% produced similar results.

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