Resampling methods on inference of the smoothness of a time series

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Abbreviated abstract: A stock market index with large ups and downs can pull back investments given its more oscillating and thus uncertain behavior. In Ferreira and Ferreira (2021) an index was introduced to ascertain the smoothness of the trajectory of a series. In the stationary case, it is the bivariate coefficient of tail dependence of Sibuya (1960). This work focuses on an inferential analysis of the smoothness index, based on resampling techniques.

References:

– H. Ferreira *& M. Ferreira*, TEST 30:198–210 (2021)
– M. Sibuya, Ann Inst Stat Math 11:195–210 (1960)

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Smoothness coefficient:

$$S_{n,m} = 1 - \lim_{u \uparrow 1} \frac{E\left(\sum_{i=n}^{m} \sum_{j \in V(i)} \mathbb{I}_{\{F_i(X_i) \leq u < F_j(X_j)\}} \mid \sum_{i=n}^{m} \mathbb{I}_{\{F_i(X_i) > u\}} > 0\right)}{E\left(\sum_{i=n}^{m} \sum_{j \in V(i)} \mathbb{I}_{\{F_j(X_j) > u\}} \mid \sum_{i=n}^{m} \mathbb{I}_{\{F_i(X_i) > u\}} > 0\right)}$$

Idea: Existing, at least, one exceedance of a high threshold between time instants *n* and *m (*i.e., $\{F_j(X_j) > u\}$*)*, the expected total of oscillations will be closer of the expected total of exceedances in processes with more oscillating trajectories.

$S_{n,m}$ ∈[0, 1]-> proportion of exceedances that are oscillations; increases with the concordance of rvs.

$$S_{n,m} = \frac{1}{m-n+1} \sum_{i=n}^{m} \frac{\lambda(i+1|i) + \lambda(i|i-1)}{2}$$

If $\lambda(j|i) = \lim_{u \uparrow 1} P\left(F_j(X_j) > u \middle| F_i(X_i) > u\right)$ exists for all $n \leq i \leq m$ e $j = i-1, i+1$.

Under stationarity, $\lambda(i+1|i)$= $\lambda(i|i-1)$= λ applied to $(X_1, X_2) \Rightarrow$ **$S_{n,m} \equiv S = \lambda$** ➡ TDC

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Estimation: 2 methods

- Generalized Jacknife – reduced bias
- Block bootstrap

$$\hat{S} := \hat{\lambda} \approx \hat{\lambda}(k) = 1 - \frac{U(k)}{k}$$

$U(k)$ - n. of upcrossings of k^{th} upper order statistic of $\{F(X_n)\}_{n\geq 1}$

Rem.: $\frac{U(k)}{k}$ corresponds to Nandagopalan estimator, $\hat{\theta}^N(k)$, of the extremal index (extreme values clustering index of time series; Nandagopalan 1990)

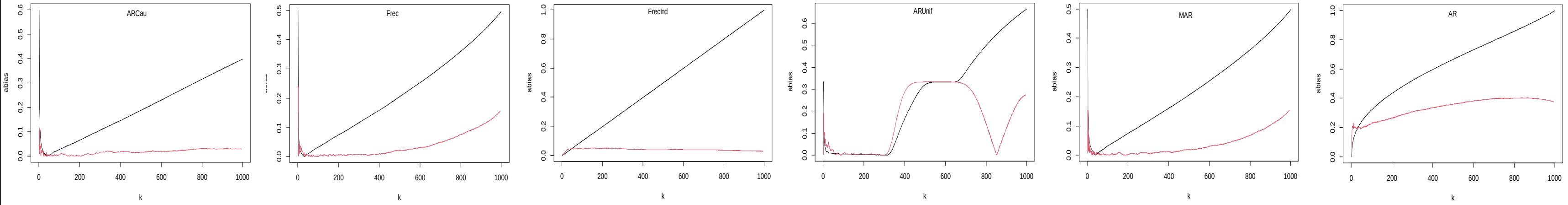
Based on Gomes et al. (2008) **reduced bias Generalized Jackknife** estimator of $\hat{\theta}^N(k)$, we consider

$$\hat{S}^{GJ}(k) := \hat{\lambda}^{GJ}(k) = 5\hat{\lambda}(\lfloor k/2 \rfloor + 1) - 2\{\hat{\lambda}(\lfloor k/4 \rfloor + 1) + \hat{\lambda}(k)\}$$

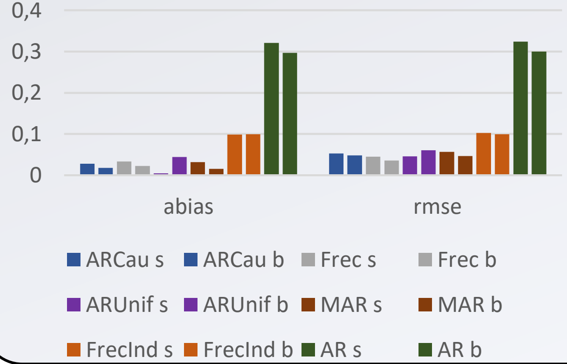
Block bootstrap on $\hat{\lambda}$ in order to preserve serial dependence: Block-Length selection method in Patton, Politis, and White (2009); function `pwsd` in R package `blocklength`

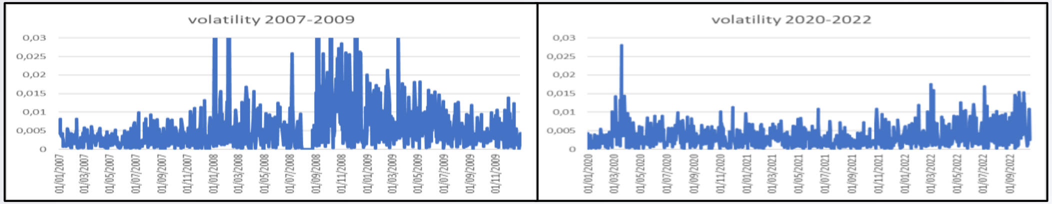
Results and Conclusions

- Reduction of bias with GJ
- Slight reduction of bias and rmse under bootstrap

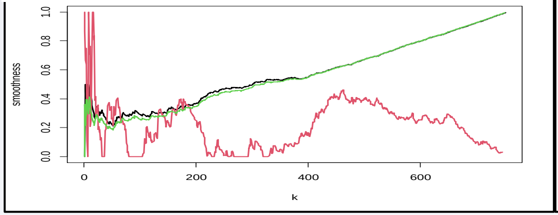


absolute bias and rmse of estimator (simple) and bootstrapped





EUR/USD: 2007-2009; $\hat{S} \cong 0.3$



EUR/USD: 2020-2022; $\hat{S} \cong 0$