

Gamma Mixture of Chi-Square Distribution: Properties, Estimation and Simulation

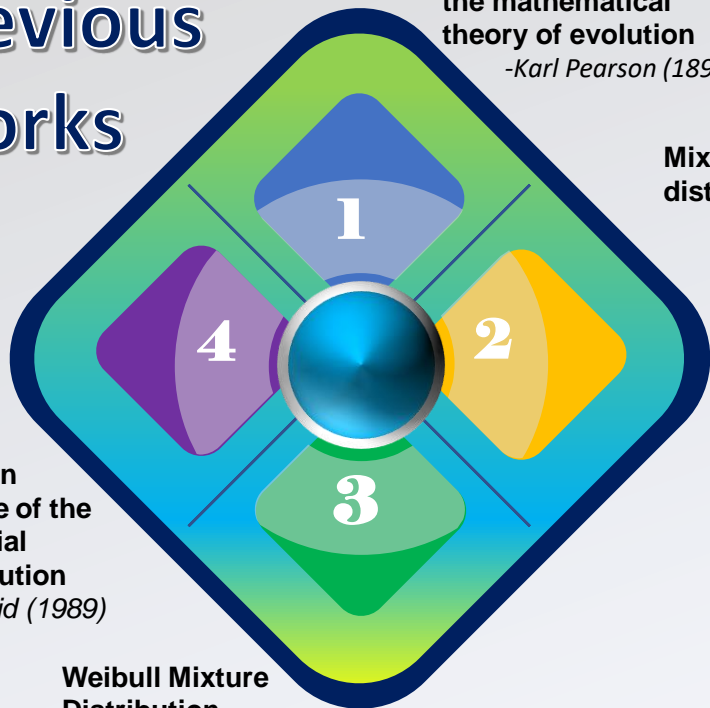
*Junar T. Lingo*¹, *Milburn O. Macalos*²
Caraga State University, Ampayon, Butuan City, Philippines

Abbreviated abstract: In this paper, we define a Gamma mixture of Chi-Square distribution using Zaman et al. . We derive the r -th raw moment for this distributions in establishing its properties. The estimation of parameters of the Gamma mixture of Chi-Square distribution was presented using the method of moments. Afterward, we simulated a random sample from the Gamma distribution and fit the samples of $n = 30$ and $n = 50$ to the Gamma mixture of Chi-Square distribution.



Previous Works

Contributions to the mathematical theory of evolution
-Karl Pearson (1894)



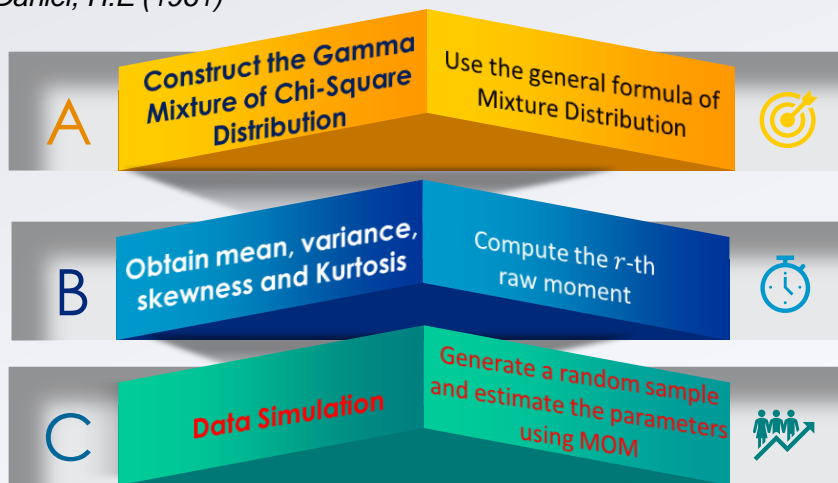
Poisson Mixture of the Binomial Distribution
-David (1989)

Weibull Mixture Distribution
-Adnan (2012)



Challenges and Solutions

Mixtures of geometric distributions
-Daniel, H.E (1961)



Methods



Obtain the Properties of the Gamma Mixture of Chi-Square Distribution

This can be done by computing the r -th raw moment defined as

$$E(X^r) = \int_{-\infty}^{\infty} f_X(x) dx$$



Fit the Gamma Mixture of Chi-Square Distribution to the Gamma Random Sample

We estimate the parameters of the Gamma Mixture of Chi-Square distribution via method of moments (MOM) and compare the fitting results with Gamma distributions using the sum square error

01

Construct Gamma Mixture of Chi-Square Distribution

Use the formula

$$f_X(x) = \int_{\theta} f(x; \theta) g(\theta) d\theta$$

with f as the Gamma distribution and g as the Chi-Square distribution



02

03

Simulate a Random Sample from the Gamma Distribution

The process can be done by using the `Statistics[RandomVariable]` and `Statistics[Sample]` command under `Statistics` package in MapleSoft. We generate a sample of 30 and 50 observations.



04



Results

$$n = 30, SSE = 9 \times 10^{-6}$$
$$n = 50, SSE = 0.0001$$

Gamma Mixture of Chi-Square

$$n = 30$$
$$\mu = 728.9$$
$$\sigma^2 = 13586$$
$$\beta_1 = 1.9$$
$$\beta_2 = 7.8$$

Comparison

$$n = 30$$
$$SSE = 0.03$$
$$n = 50$$
$$SSE = 0.01$$

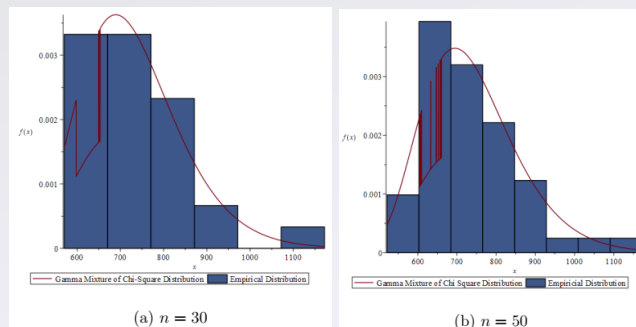
$$n = 50$$
$$\mu = 739$$
$$\sigma^2 = 14909$$
$$\beta_1 = 0.1$$
$$\beta_2 = 2$$

$$n = 30$$
$$\mu = 755$$
$$\sigma^2 = 13214$$
$$\beta_1 = 0.3$$
$$\beta_2 = 3.1$$

Gamma

$$n = 50$$
$$\mu = 755$$
$$\sigma^2 = 13214$$
$$\beta_1 = 0.3$$
$$\beta_2 = 3.1$$

Fitted Gamma Mixture of Chi-Square Distributions



Conclusions



The Gamma Mixture distribution is Leptokurtic



The Gamma mixture distribution is skewed to the right



The GMCS distribution outscore Gamma distribution in fitting

