BURR X EXPONENTIAL WEIBULL DISTRIBUTION: PROPERTIES AND APPLICATION

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Abbreviated Abstract. A new distribution called Burr X Exponential Weibul distribution (BXE-W) is developed. The characteristics features contain in this new distribution (BXE-W) aids to boost its flexibility. The respective density and distribution functions of this new distribution (BXE-W) were derived. Simulation study shows that the estimated parameters of BXE-W are consistent as the BIAS and RMSE approach zero. Finally, real data sets were used to validate the results obtained from MLE method. The results show that Burr X Exponential Weibull (BXE-W) distribution best fit the data sets compare to the competitive distributions used in this study. Perhaps, this new distribution may be useful to model positive real life data sets that may possess the characteristics feature of Burr X, Exponential and Weibull distributions.

Introduction and Methodology

Weibull distribution has been extended by various families of distributions as aimed to improve on its flexibility in terms of capturing the non-monotone hazard rate function.Therefore, a new distribution called Burr X Exponential Weibull (BXE-W) distribution is developed with its cdf and pdf defined below:

$$F\left[x;\theta,\lambda,\phi,\gamma\right]=\left[1-exp\left\{-\left(exp\{\lambda\left(exp\{\frac{x}{\gamma}\}^{\phi}-1\right)\}-1\right)^{2}\right\}\right]^{\theta} \tag{1}$$

$$f\left[x;\theta,\lambda,\beta\right]=\frac{2\theta\lambda\frac{\phi}{\gamma}\left(\frac{x}{\gamma}\right)^{\phi-1}exp\{\lambda\left(exp\{\frac{x}{\gamma}\}^{\phi}-1\right)\}}{exp-\{\frac{x}{\gamma}\}^{\phi}}$$
$$\times exp\left\{-\left(exp\{\lambda\left(exp\{\frac{x}{\gamma}\}^{\phi}-1\right)\}-1\right)^{2}\right\}\left(exp\{\lambda\left(exp\{\frac{x}{\gamma}\}^{\phi}-1\right)\}-1\right)$$
$$\times\left[1-exp\left\{-\left(exp\{\lambda\left(exp\{\frac{x}{\gamma}\}^{\phi}-1\right)\}-1\right)^{2}\right\}\right]^{\theta-1} \tag{2}$$

Simulation

From table 1 below, the four Parameters of Burr X Exponential Weibul (BXE-W) Distribution are consistence as mean approaches initial values (0.7, 0.4, 0.6, 0.3) ; bias and root mean-squared error (RMSE) approach zero

Table: 1: Means, Bias and RMSE for the BXE-W parameter estimates when $\theta = 0.7, \lambda = 0.4, \alpha = 0.6, \varphi = 0.3$.

		MLE			
sample size		Estimated parameters			
n	Evaluators	P1=θ	P2=λ	P3=ϕ	P4=γ
	MEAN	0.7006	0.3718	0.8001	0.2885
10	*BIAS*	0.0006	-0.0282	0.2001	-0.0115
	RMSE	0.3206	0.1115	0.3739	0.0646
	MEAN	0.7026	0.3946	0.7205	0.3012
20	*BIAS*	0.0026	-0.0054	0.1205	0.0012
	RMSE	0.2760	0.0833	0.2691	0.0475
	MEAN	0.7049	0.4046	0.6699	0.3065
40	*BIAS*	0.0049	0.0046	0.0699	0.0065
	RMSE	0.2376	0.0531	0.1994	0.0361
	MEAN	0.7038	0.4072	0.6542	0.3084
60	*BIAS*	0.0038	0.0072	0.0542	0.0084
	RMSE	0.2050	0.0451	0.1711	0.0310
	MEAN	0.7094	0.4086	0.6322	0.3077
100	*BIAS*	0.0094	0.0086	0.0322	0.0077
	RMSE	0.1690	0.0346	0.1297	0.0245

Application to data set

The data sets represents the remission times (in months) of a random sample of 128 bladder cancer patients previously used by Lee and Wang (2003). The result from the data set is in the table 2 below; thus, the distribution with the lowest Log Likelihood (LL) and Akaike Information Criteria (AIC) values is Burr X Exponential - Weibull (BXE-W) distribution compares to other competitive distributions such as Topp Leone Exponential Weibull (TLE-W), Transmuted Weibull (TW), Weibull (W), Burr X Exponential Lomax (BXE-L) and Exponentiated Generalized Weibull (ET-GW). Therefore, BXE-W best fits the data sets

Table: 5a: ML Estimates on Data set 1

Models	θ	σ	λ	ϕ	γ	α	LL	AIC	R
BXE-W	6.4605	-	0.4750	0.0929	2.8168	-	410.754	829.508	1
TLE-W	-	7.6289	0.6112	0.2638	0.4381	-	410.933	829.8660	2
TW	-	1.1337	-	14.6183	0.7440	-	411.9584	829.9168	3
W	-	-	-	1.0477	9.5613	-	414.0869	832.1738	4
BXE-L	0.4978	-	12.9334	-	0.0295	3.2198	424.2681	856.5362	5
ET-GW	-	1.0537	1.3702	0.2765	0.4993	-	540.2794	1088.559	6